

B Meson Decay Constants Using NRQCD

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Recent results for B meson decay constants with NRQCD b-quarks and clover light quarks are discussed. Perturbative matching factors through $O(\alpha/M)$ are now available and incorporated into the analyses. An $O(\alpha a)$ improvement term to the heavy-light axial current is identified and included. The slope of $f_{PS}\sqrt{M_{PS}}$ versus $1/M_{PS}$ is significantly reduced by these corrections.

1. Introduction

The heavy-light pseudoscalar meson decay constant, f_{PS} , is defined through the matrix element of the heavy-light axial vector current between the pseudoscalar meson state and the hadronic vacuum. In Euclidean space one has,

$$\langle 0 | A_\mu | PS \rangle = p_\mu f_{PS}. \quad (1)$$

f_{D_s} has been measured experimentally, giving values consistent with lattice predictions. f_B , on the other hand is unlikely to be measured directly any time soon via leptonic B decays, $B \rightarrow l \bar{\nu}$. This fact coupled with the importance of f_B in analyses of $B_0 \bar{B}_0$ mixing phenomena, makes an accurate lattice determination of f_B particularly relevant [1].

This talk describes work being carried out by A. Ali Khan, T. Bhattacharya, S. Collins, C. Davies, R. Gupta, C. Morningstar, U. Heller, J. Sloan and myself, on heavy meson decay constants with NRQCD b-quarks. Earlier works exist by C. Davies [2], Draper & McNeile [3] and S. Hashimoto [4]. Dr. Onogi discusses the Hiroshima group's results on NRQCD decay constants in a separate talk [5].

New developments since Lattice '96 include,

- Completion of the one-loop matching calculation between lattice NRQCD and continuum full QCD axial currents through $O(\alpha/M)$.

- Inclusion of an $O(\alpha a)$ discretization correction to the local heavy-light axial current.
- Considerable decrease in the slope of $f_{PS}\sqrt{M_{PS}}$ versus $1/M_{PS}$ once the discretization correction and the matching Z-factors have been included. This makes the connection between the static limit and the physical b-quark region much smoother than previously thought.
- Higher statistics on quenched calculations at $\beta = 6.0$ that include $1/M^2$ corrections to the action and currents at tree level. This latter data is still being analyzed and I will only show older data in this talk with corrections through $O(1/M)$.

2. The Action and Current Operators

We use the NRQCD action to simulate b-quarks [6,7]. It is given by (in continuum notation),

$$\bar{\psi}(\mathbf{D}_t + H_0 + \delta H)\psi, \quad (2)$$

We have dropped the rest mass term and H_0 is the nonrelativistic kinetic energy operator,

$$H_0 = -\frac{D^{(2)}}{2M_0}, \quad (3)$$

and through $O(\Lambda_{QCD} (\Lambda_{QCD}/M))$ one has

$$\delta H = -\frac{g}{2M_0} \boldsymbol{\sigma} \cdot \mathbf{B} \quad (4)$$

In our more recent simulations we have added the terms

$$\frac{ig}{8(M_0)^2} (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) - \frac{g}{8(M_0)^2} \boldsymbol{\sigma} \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})$$

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which are of $O(\Lambda_{QCD} (\Lambda_{QCD}/M)^2)$ and

$$-\frac{(D^{(2)})^2}{8(M_0)^3} \quad (5)$$

which we believe is the dominant $O(\Lambda_{QCD} (\Lambda_{QCD}/M)^3)$ contribution. These latter calculations also include discretization corrections to the lattice laplacian and the lattice time-derivative[7]. All operators in the NRQCD action and in the heavy-light currents discussed below, are tadpole improved ($U \rightarrow U/u_0$ with $u_0 =$ fourth root of the average plaquette). Tree level coefficients are used in the action.

For light quarks we have data using the Wilson and the $C_{SW} = 1$ clover quark actions. The bulk of our results, however, come from the tadpole improved clover action. Gauge fields were created using the Wilson plaquette action. The quenched configurations are at $\beta = 6.0$; one set provided by the UKQCD collaboration and another newer set created by us on the LANL and NCSA CM5 machines. The dynamical calculations were carried out on the HEMCGC $n_f = 2$ staggered configurations at $\beta = 5.6$.

Heavy-light currents in full QCD have the form $\bar{q}\Gamma h$. The four component Dirac spinor for the heavy quark, h , is related to the two component NRQCD heavy quark (heavy anti-quark) fields, ψ ($\bar{\psi}$), via an inverse Foldy-Wouthuysen transformation.

$$h = U_{FW}^{-1} \Psi_{FW} = U_{FW}^{-1} \begin{pmatrix} \psi \\ \tilde{\psi} \end{pmatrix} \quad (6)$$

$$\begin{aligned} U_{FW}^{-1} &= 1 - \frac{1}{2M}(\boldsymbol{\gamma} \cdot \mathbf{D}) \\ &+ \frac{1}{8M^2}(D^{(2)} + g\boldsymbol{\Sigma} \cdot \mathbf{B} - 2i\boldsymbol{\alpha} \cdot \mathbf{E}) \\ &+ \mathcal{O}(1/M^3) \end{aligned} \quad (7)$$

$\boldsymbol{\alpha} \equiv \gamma_0 \boldsymbol{\gamma}$ and $\boldsymbol{\Sigma} = \text{diag}(\boldsymbol{\sigma}, \boldsymbol{\sigma})$. All our expressions have been converted to Euclidean space with hermitian γ -matrices.

Through $O(1/M)$ one has at tree-level,

$$\begin{aligned} J &= J^{(0)} + J^{(1)} \\ &= \bar{q}\Gamma Q - \frac{1}{2M}\bar{q}\Gamma(\boldsymbol{\gamma} \cdot \mathbf{D})Q \end{aligned} \quad (8)$$

$$Q = \frac{1}{2}(1 + \gamma_0)\Psi_{FW}.$$

In the next section we discuss what happens at one-loop.

3. Perturbative Matching

In order to match between lattice currents used in our simulations and those of continuum QCD, we consider the process in which a heavy quark of momentum p is scattered by the heavy-light current into a light quark of momentum p' . For a one-loop matching we need to go through the following steps.

1. Carry out the one-loop calculation for the above process in full QCD.
2. Expand the amplitude in terms of p/M , p'/M etc.
3. Identify operators in the effective theory (NRQCD) that would reproduce these $1/M$ corrections.
4. Carry out a one-loop mixing matrix calculation in the effective theory.

For HQET analogous calculations have been done by Eichten & Hill [8], Golden & Hill [9], and Neubert [10]. Colin Morningstar and I have now completed the matching calculation, between lattice NRQCD and full continuum QCD, for the time component of the axial vector current through $O(\alpha/M)$ [11].

Continuum calculation

A one-loop calculation in full QCD finds that $\langle q(p') | A_0 | h(p) \rangle_{QCD}$ involves terms proportional to,

$$\begin{aligned} &\bar{u}_q(p')\gamma_5\gamma_0 u_h(p) \quad , \quad \frac{p_0}{M} \bar{u}_q(p')\gamma_5 u_h(p) \\ &\frac{p \cdot p'}{M^2} \bar{u}_q(p')\gamma_5\gamma_0 u_h(p) \quad , \quad \frac{p'_0}{M} \bar{u}_q(p')\gamma_5 u_h(p) \\ &\frac{p \cdot p'}{M^2} \frac{p_0}{M} \bar{u}_q(p')\gamma_5 u_h(p) \quad + \quad \mathcal{O}(1/M^2) \end{aligned} \quad (9)$$

We set the light quark mass equal to zero, i.e. we ignore terms such as m_q/M .

The above terms can be reproduced via three operators in the effective theory, $J_A^{(i)} = \bar{q} O_A^{(i)} Q$, $i = 0, 1, 2$, after making use of equations of motion for the light quark.

$$\begin{aligned} J_A^{(0)} &= \bar{q} \gamma_5 \gamma_0 Q \\ J_A^{(1)} &= -\frac{1}{2M} \bar{q} \gamma_5 \gamma_0 (\boldsymbol{\gamma} \cdot \mathbf{D}) Q \\ J_A^{(2)} &= \frac{1}{2M} (\mathbf{D} \bar{q} \cdot \boldsymbol{\gamma}) \gamma_5 \gamma_0 Q \end{aligned} \quad (10)$$

$J^{(0)}$ and $J^{(1)}$ coincide with eq.(8), $J^{(2)}$ appears only at one-loop. So, through one-loop $\langle q(p') | A_0 | h(p) \rangle_{QCD}$ can be written as,

$$\begin{aligned} \langle q(p') | A_0 | h(p) \rangle_{QCD} &= \sum_{i=0}^2 \eta_i^A \langle J_A^{(i)} \rangle_{ren} \\ &\equiv \sum_{i=0}^2 \eta_i^A \left(\bar{u}_q(p') \tilde{O}_A^{(i)} U_Q(p) \right) \end{aligned} \quad (11)$$

$\tilde{O}_A^{(i)}$ are momentum space representations of the operators between quark fields in eq.(10). We have calculated the one-loop coefficients, η_i^A , using dimensional regularization (with totally anti-commuting γ_5) in the \overline{MS} scheme. A gluon mass, λ , is introduced as an infrared regulator. We find,

$$\begin{aligned} \eta_0^A &= 1 + \frac{\alpha}{3\pi} \left[3 \ln \frac{M}{\lambda} - \frac{3}{4} \right] \\ \eta_1^A &= 1 + \frac{\alpha}{3\pi} \left[3 \ln \frac{M}{\lambda} - \frac{19}{4} \right] \\ \eta_2^A &= \frac{\alpha}{3\pi} \left[12 - \frac{16\pi}{3} \frac{M}{\lambda} \right] \end{aligned} \quad (12)$$

The infrared divergent terms will cancel when we match to a lattice regularized one-loop calculation.

Lattice calculation

The lattice currents used in our simulations are discretized versions of the operators in eq.(10). In the absence of any improvement, $J_{A,L}^{(0)}$ becomes

a local heavy-light current, and $J_{A,L}^{(1)}$ and $J_{A,L}^{(2)}$ have $\mathbf{D} \rightarrow \{\text{lattice symmetric covariant derivative}\}$. These lattice operators are only defined up to improvement terms. In fact, in the next section we will argue that a consistent one-loop calculation with clover (i.e. $O(a)$ improved) light quarks, requires that an $O(\alpha a)$ correction term be added to the local $J_{A,L}^{(0)}$.

The lattice matrix elements $\langle J_{A,L}^{(i)} \rangle$ can be related to the $\langle J_A^{(i)} \rangle_{ren}$ on the RHS of eq.(11) via a mixing matrix \mathbf{Z} .

$$\langle J_{A,L}^{(i)} \rangle = \sum_j Z_{ij} \langle J_A^{(j)} \rangle_{ren} \quad (13)$$

From (11) and (13) one extracts the matching relation between matrix elements in full QCD and those evaluated in lattice simulations,

$$\begin{aligned} \langle A_0 \rangle_{QCD} &= \sum_{i,j} \eta_i^A Z_{ij}^{-1} \langle J_{A,L}^{(j)} \rangle \\ &\equiv \sum_j C_j \langle J_{A,L}^{(j)} \rangle \end{aligned} \quad (14)$$

$\vec{C} = \vec{\eta}^A \mathbf{Z}^{-1}$ is the vector of matching coefficients for this problem.

We have calculated the one-loop contributions to Z_{ij} in lattice perturbation theory for lattice actions including H_0 and δH of eq.(4). Equation (14) can be expanded out as,

$$\begin{aligned} \langle A_0 \rangle_{QCD} &= \\ &\left(1 + \alpha \left[B_0 - \frac{1}{2}(C_q + C_Q) - \zeta_{00} - \zeta_{10} \right] \right) \langle J_{A,L}^{(0)} \rangle \\ &+ \left(1 + \alpha \left[B_1 - \frac{1}{2}(C_q + C_Q) - \zeta_{01} - \zeta_{11} \right] \right) \langle J_{A,L}^{(1)} \rangle \\ &+ \alpha \left[B_2 - \zeta_{02} - \zeta_{12} \right] \langle J_{A,L}^{(2)} \rangle + \mathcal{O}(\alpha^2) \end{aligned} \quad (15)$$

The B_i 's come from the η_i^A 's in (12) and C_q and C_Q are the light and heavy quark lattice wave function renormalizations respectively. The ζ_{ij} are the one-loop vertex correction contributions to Z_{ij} . Before proceeding with combining perturbative numbers with simulation results for the matrix elements $\langle J_{A,L}^{(i)} \rangle$, we must discuss improvement of the local current $J_{A,L}^{(0)}$.

4. An $O(\alpha a)$ Correction to the Heavy-Light Axial Current

There has been a lot of work recently on improving quark bilinear operators such as the vector and axial currents. For the light quark sector, the DESY [12] group has shown that an improved axial vector current takes on the form (suppressing isospin indices),

$$A_\mu^I = A_\mu^{loc} + c_A a D_\mu P \quad (16)$$

with P the pseudoscalar density and D_μ the symmetric lattice derivative. In perturbation theory, c_A starts out at $O(\alpha)$ [12,13]. Less is known about improvement of the local heavy-light or static-light axial currents. Borrelli & Pittori [14] have shown that through one-loop, static-light bilinears using clover light quarks are free of $O(a)$ and $O(\alpha a \log(a))$ terms. They did not consider $O(\alpha a)$ terms.

Our one-loop calculation of the mixing matrix Z_{ij} shows that there is an $O(\alpha a)$ lattice artifact term whose effects can be removed by improving $J_{A,L}^{(0)}$. The improved heavylight lattice axial current acquires a correction term that is the precise analogue of the correction term in eq.(16). The argument goes as follows.

In order to calculate $Z_{02} = \alpha \zeta_{02}$ one needs to start from $\langle J_{A,L}^{(0)} \rangle$ and project out terms proportional to $\langle J_A^{(2)} \rangle_{ren} = \frac{1}{2M} \bar{u}_q(-i\mathbf{p}' \cdot \boldsymbol{\gamma}) \gamma_5 \gamma_0 U_Q$. After calculating ζ_{02} in this way, we find that ζ_{02} has a term that grows with aM , the dimensionless heavy quark mass. So,

$$\alpha \zeta_{02} \langle J_A^{(2)} \rangle_{ren} = \alpha (aM A_1 + A_2) \langle J_A^{(2)} \rangle_{ren}$$

has an $O(\alpha a)$ term with the structure,

$$\alpha a \bar{u}_q(-i\mathbf{p}' \cdot \boldsymbol{\gamma}) \gamma_5 \gamma_0 U_Q.$$

One can interpret this term as coming from a lattice operator,

$$J_{A,L}^{(3)} = a (\mathbf{D}\bar{q} \cdot \boldsymbol{\gamma}) \gamma_5 \gamma_0 Q \quad (17)$$

and what we are finding is,

$$\begin{aligned} \alpha \zeta_{02} \langle J_A^{(2)} \rangle_{ren} \\ = \alpha \zeta_{02}^{true} \langle J_A^{(2)} \rangle_{ren} + \alpha \zeta_{03} \langle J_{A,L}^{(3)} \rangle_{ren} \end{aligned} \quad (18)$$

and from eq.(13),

$$\begin{aligned} \langle J_{A,L}^{(0)} \rangle &= \sum_{j=0,1,2} Z_{0j} \langle J_A^{(j)} \rangle_{ren} \\ &\quad + \alpha \zeta_{03} \langle J_{A,L}^{(3)} \rangle_{ren} \end{aligned} \quad (19)$$

with $Z_{02} = \alpha \zeta_{02}^{true}$. If one wants to have the same set of matrix elements on the RHS of eq.(19) as in the continuum theory, the last term must be removed. This is easily accomplished by improving $J_{A,L}^{(0)}$.

$$J_{A,L}^{(0)} \rightarrow J_{A,L}^{(0),I} = J_{A,L}^{(0)} - \alpha \zeta_{03} J_{A,L}^{(3)} \quad (20)$$

and one now has,

$$\langle J_{A,L}^{(0),I} \rangle = \sum_{j=0,1,2} Z_{0j} \langle J_A^{(j)} \rangle_{ren} \quad (21)$$

Going through the same steps that led from (13) to (14) and then to (15), one ends up with an equation very similar to eq.(15) with $\langle J_{A,L}^{(0)} \rangle \rightarrow \langle J_{A,L}^{(0),I} \rangle$ and $\zeta_{02} \rightarrow \zeta_{02}^{true}$.

Although $J_{A,L}^{(2)}$ and $J_{A,L}^{(3)}$ are proportional to each other, $J_{A,L}^{(3)} = 2 aM J_{A,L}^{(2)}$, they play very different roles. $J_{A,L}^{(2)}$ is the lattice version of a current operator that exists in the continuum theory. It is a $1/M$ correction to the static heavy-light axial current and is absent in the static theory. $J_{A,L}^{(3)}$, on the other hand, has no continuum counter part. It survives into the lattice static theory. So the static limit of eq.(15) becomes,

$$\begin{aligned} \langle A_0 \rangle_{QCD} \\ = (1 + \alpha [B_0 - \frac{1}{2}(C_q + C_Q) - \zeta_{00}]) \langle J_{A,L}^{(0),I} \rangle_{stat} \\ = (1 + \alpha [B_0 - \frac{1}{2}(C_q + C_Q) - \zeta_{00}]) \langle J_{A,L}^{(0)} \rangle_{stat} \\ - \alpha \zeta_{03} \langle J_{A,L}^{(3)} \rangle_{stat} + O(\alpha^2) \end{aligned} \quad (22)$$

We will see in the next section that the last correction term in (22) significantly reduces the value of $f_{PS}\sqrt{M_{PS}}$ in the static theory.

It is easy to see that the improvement term $J_{A,L}^{(3)}$ is of the same form as the second term in eq.(16). If one defines a heavylight pseudoscalar density $P_{HL} \equiv \bar{q}\gamma_5 Q$, then

$$\begin{aligned}
\alpha a D_0 P_{HL} &= \alpha a (D_0 \bar{q}) \gamma_5 Q + \mathcal{O}(\alpha a/M) \\
&\approx -\alpha a (D_0 \bar{q} \gamma_0) \gamma_5 \gamma_0 Q \quad (23)
\end{aligned}$$

In the above we ignore the term coming from the time-derivative acting on the heavy quark field Q , since equations of motion make that into an $\mathcal{O}(\alpha a/M)$ term and we neglect such contributions together with contributions of $\mathcal{O}(\alpha/M^2)$. Applying light quark equations of motion to the last expression in (23) gives the operator $J_{A,L}^{(3)}$ of eq.(17) multiplied by α .

5. Some Quenched f_B Results

An analysis of NRQCD heavy meson decay constants including one-loop matching factors, has been carried out for the first time in Ref.[15]. The data was obtained on $16^3 \times 48$ quenched configurations at $\beta = 6.0$. Both the gauge configurations and light propagators were generously provided by the UKQCD collaboration. Tadpole improved clover light propagators were used. The NRQCD action included the δH of eq.(4) but no $1/M^2$ terms. Two light κ values around the strange quark mass and four NRQCD bare heavy quark masses were used. We also have results for static heavy quarks.

For the perturbation theory, we use α_V of Lepage&Mackenzie[16]. The q^* for this matching calculation has not been calculated yet. So, we present results for both $q^* = 1/a$ and $q^* = \pi/a$. The $\log(aM)$ terms that appear in the matching coefficients after cancellation of logarithmic IR divergences, are set to a constant value $\log(a\bar{m}_b)$ for all bare heavy quark masses. $\bar{m}_b = 4.1(1)GeV$ is the b-quark \overline{MS} mass at a scale equal to its value [17]. Alternatively we could have used the one-loop renormalization group improved expression with an overall factor of $(\alpha(\bar{m}_b)/\alpha(a^{-1}))^{-2/\beta_0}$ for the two matching coefficients C_0 and C_1 . The difference between the two approaches is very slight and at the B-meson numerically undetectable. Since we presently have results at only one lattice spacing, we have used the simpler $\log(a\bar{m}_b)$ prescription. Details of the simulations and fitting procedures are given in Ref.[15]. Figure 1. shows

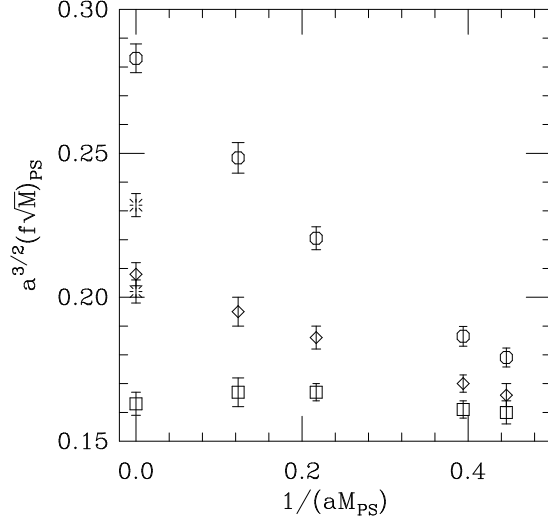


Figure 1. Heavy-Light Decay Constants at κ_s . Circles : tree-level ; Diamonds : one-loop matching with $aq^* = \pi$; Squares : one-loop matching with $aq^* = 1$. We demonstrate the effect of the correction term to $J_{A,L}^{(0)}$, by showing the one-loop matched result in the static limit without the last term in eq.(22) for $aq^* = \pi$ (upper burst) and $aq^* = 1$ (lower burst). The physical B_s meson is just below $1/aM_{PS} = 0.4$.

$a^{3/2} f_{PS} \sqrt{M_{PS}}$ versus $1/aM_{PS}$ for $\kappa = \kappa_s$. We show the tree-level results and the one-loop results for the two q^* 's. For the static limit we also indicate the one-loop values without the $\langle J_{A,L}^{(3)} \rangle$ correction term. The physical B_s meson is just below $1/aM_{PS} = 0.4$ on the figure.

There are several interesting things to note in Figure 1.

1. After including the one-loop matching factors and improvement of $J_{A,L}^{(0)}$, the slope of $a^{3/2} f_{PS} \sqrt{M_{PS}}$ versus $1/aM_{PS}$ as one leaves the

static limit decreases considerably. For $q^* = 1/a$ it is consistent with zero. For $q^* = \pi/a$ the relative slope (the slope divided by $(f_{PS}\sqrt{M_{PS}})_{stat}$) is $\sim -1\text{GeV}$. Unfortunately a precise value of q^* is required in order to make a more quantitative statement. Ref.[18] quotes $q^* = 2.18/a$ for the static theory.

2. Around the physical B meson region the one-loop corrections are a 9 - 13% effect depending on q^* at κ_s . The difference due to q^* of $1/a$ or π/a is a $\sim 5\%$ effect on the final answer. This is one measure of the uncertainty in f_{B_s} coming from higher orders in the matching calculation.

The static limit is much more sensitive to q^* and the one-loop corrections are large (25 - 40% at κ_s and 10 - 30% at κ_c), with a third of the shift coming from the $O(\alpha a)$ correction to $J_{A,L}^{(0)}$.

We use a scale of $a^{-1} = 2.0(2)\text{GeV}$ to convert to physical units. In a quenched calculation, different observables can lead to different a^{-1} 's. We have used an a^{-1} consistent with quenched light quark calculations (M_ρ , f_π etc.) and allow for a large error of 10%. This feeds back into one of the dominant systematic errors in our final estimate for f_B . We find at κ_s ,

$$f_{B_s} = \begin{cases} 0.198(8)(30)(17) \text{ GeV} & q^* = 1/a \\ 0.209(8)(32)(17) \text{ GeV} & q^* = \pi/a \end{cases}$$

and after extrapolating to κ_c ,

$$f_B = \begin{cases} 0.174(28)(26)(16) \text{ GeV} & q^* = 1/a \\ 0.183(32)(28)(16) \text{ GeV} & q^* = \pi/a \end{cases}$$

Errors in the first brackets correspond to statistical plus fitting plus κ extrapolation errors. In the second brackets we give a^{-1} systematic errors and the third brackets summarize our estimates for higher order perturbative and $1/M^2$ corrections. We now have more recent simulations with $1/M^2$ terms included at tree level both in the NRQCD action and the currents (see next section on Work in Progress). Their contributions are at the 3 - 4% level. The Hiroshima group reports similar findings for the $1/M^2$ current corrections [5]. We have not included any estimate for continuum extrapolation corrections, since at the moment we only have results at a single value

of β . Studies with Wilson and/or static fermions [19–21] have shown noticeable lattice spacing dependence around $\beta = 6.0$. It will be interesting to do a thorough scaling study with clover light fermions and an improved $J_{A,L}^{(0)}$.

6. Work in Progress and Future Plans

Two sets of NRQCD heavy meson decay constant data are currently being analyzed by my collaborators.

1. The Glasgow-LANL-OSU-Kentucky (GLOK) collaboration has results on $16^3 \times 48$ quenched configurations at $\beta = 6.0$ [22]. Both the NRQCD action and the current operators include $1/M^2$ corrections at tree level. We also use a better time evolution equation to obtain heavy propagators (which follows from a slightly modified lattice NRQCD action) than in the simulations of the previous section. The new evolution equation eliminates a residual $O(a\Lambda(\Lambda/M))$ error in amplitudes. The light quark action employed is the tadpole-improved clover action. We use five κ values and six heavy quark masses. Perturbative matching calculations for the new lattice action have now also been completed [11]. Some preliminary tree-level results were presented at St. Louis [23].

2. The SCRI-Glasgow-OSU (SGO) collaboration has results on the HEMCGC $n_f = 2$, $am_{dyn} = 0.01$ dynamical staggered configurations at $\beta = 5.6$. Both Wilson and tadpole-improved clover light quarks have been combined with NRQCD heavy quarks. The NRQCD action and the heavy-light currents were corrected through $O(1/M)$. Three κ values and eleven heavy quark mass values were used. The Wilson light fermion results are published in Ref.[24]. Tree level analyses of the clover light quark data have been reported on in [25]. We are now completing the analysis including one-loop matching [26].

Several matching coefficient projects are also on the agenda. The one-loop matching calculation for the heavy-light vector current is well underway and q^* 's for both axial and vector currents

are high on our list. We have also started studies of nonperturbative renormalization of NRQCD operators.

Finally, quenched simulations at $\beta = 5.7$ have begun and in the future we plan to go onto $\beta = 6.2$.

7. Summary

The NRQCD approach to B meson decays is looking very promising. The $1/M$ expansion is working well and appears to be under good control. The first one-loop perturbative matching calculations have been completed and incorporated into our analyses. Uncertainties due to higher orders in perturbative matching are estimated to be at the $\sim 5\%$ level around the B meson. Many more results should be forthcoming soon, including the use of an improved heavy quark time evolution equation, and studies of $1/M^2$ corrections, unquenching and scaling.

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